

## Inertial Mass Viewed as Reaction of the Vacuum to Accelerated Motion

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### ABSTRACT:

Preliminary analysis of the momentum flux (or of the Poynting vector) of the classical electromagnetic version of the quantum vacuum consisting of zero-point radiation impinging on accelerated objects as viewed by an inertial observer suggests that the resistance to acceleration attributed to inertia may be a force of opposition originating in the vacuum. This analysis avoids the *ad hoc* modeling of particle-field interaction dynamics used previously by Haisch, Rueda and Puthoff (1994) to derive a similar result. This present approach is not dependent upon what happens at the particle point but on how an external observer assesses the kinematical characteristics of the zero-point radiation impinging on the accelerated object. A relativistic form of the equation of motion results from the present analysis

### INTRODUCTION:

It was recently proposed [by Haisch, Rueda and Puthoff(1994), henceforth HRP], that the inertial property of matter could originate in interactions between electromagnetically interacting particles at the level of their most fundamental components (e.g., electrons, quarks) and the quantum vacuum (QV). This general idea is a descendent of a conjecture of Sakharov (1968) for the case of gravity that can be extended by the principle of equivalence to the case of inertia. In the accompanying paper (Haisch and Rueda, 1997), we give more references and further discussion pertinent to this point. The approach of stochastic electrodynamics (SED) was used in HRP to study the classical dynamics of a highly idealized model of a fundamental particle constituent of matter (that contained a "parton", i.e., a surrogate for a very fundamental particle component) responding to the driving forces of the so-called classical electromagnetic zero-point field (ZPF), the classical analog of the QV.

The primary purpose of the endeavour reported here is to find a simpler approach, which attempts to avoid drawbacks and model-related issues in the approach of HRP (see Cole 1997, Cole and Rueda, 1997), by examining how an opposing flux of radiative energy and momentum should arise under natural and suitable assumptions in an accelerated frame from the viewpoint of an inertial observer and without regard to details of particle-field dynamics, i.e., independently of any dynamical models for particles. Using relativistic transformations for the electromagnetic fields, it is argued that upon acceleration a time rate of change of momentum density or momentum flux will arise out of the ZPF, and that this turns out to be directed against and linearly proportional to the acceleration. This arises after evaluation of the ZPF momentum density as it appears at a given point in an accelerated frame  $S$ , to an independent inertial laboratory observer due to transformations of the fields from the observer's inertial laboratory reference frame,  $I_*$ , to another inertial frame  $I_t$  instantaneously comoving with the object and from the viewpoint of the observer in the laboratory inertial frame  $I_*$ . Absorption or scattering of this radiation by the accelerated charged particle will thus result in a force opposing the acceleration, yielding an  $\mathbf{f} = m\mathbf{a}$  relation for subrelativistic motions. (Vectors are symbolized throughout by boldface letters or by an arrow or a line on top of the letter).

### ZERO-POINT FIELD AND HYPERBOLIC MOTION:

We assume a non-inertial frame of reference,  $S$ , accelerated in such a way that the acceleration  $\mathbf{a}$  as seen from a particle fixed to a specific point, namely  $(c^2/a, 0, 0)$ , in the accelerated system,  $S$ , remains constant. Such condition leads as in Boyer (1984) and HRP to the well-known case of hyperbolic motion (see.e.g. Rindler, 1991) We again

represent the classical electromagnetic ZPF in the traditional form and assume the same three reference systems  $I_*$ ,  $I_\tau$  and  $S$ , as in HRP and originally introduced in Boyer (1984).  $I_*$  is the inertial laboratory frame.  $S$  is the accelerated frame in which the particle is placed at rest at the point  $(c^2/a, 0, 0)$ .  $\tau$  is the particle proper time as measured by a clock located at this same particle point  $(c^2/a, 0, 0)$  of  $S$ .  $I_\tau$  is an inertial system whose  $(c^2/a, 0, 0)$  point at proper time  $\tau$  exactly coincides with the particle point of  $S$ . The acceleration of this  $(c^2/a, 0, 0)$  point of  $S$  is  $\mathbf{a}$  as measured from  $I_\tau$ . Hyperbolic motion is defined such that  $\mathbf{a}$  is the same for all proper times  $\tau$  as measured in the corresponding  $I_\tau$  frames at a point  $(c^2/a, 0, 0)$  that in each one of these  $I_\tau$  frames instantaneously comoves and coincides with the corresponding particle point, namely  $(c^2/a, 0, 0)$  of  $S$ . At proper time  $\tau = 0$ , this particle point of the  $S$  system instantaneously coincides with the  $(c^2/a, 0, 0)$  point of  $I_*$  and thus  $I_* = I_\tau (\tau = 0)$ . We refer to the observer's laboratory time in  $I_*$  as  $t_*$ , chosen such that  $t_* = 0$  at  $\tau = 0$ . For simplicity we let the particle acceleration  $\mathbf{a}$  at proper time  $\tau$  take place along the  $x$ -direction so that  $\mathbf{a} = a\hat{x}$ , is the same constant vector, as seen at every proper time  $\tau$  in every corresponding  $I_\tau$  system. The acceleration of the  $(c^2/a, 0, 0)$  point of  $S$  as seen from  $I_*$  is  $\mathbf{a}_* = \gamma_\tau^{-3} \mathbf{a}$ . We take  $S$  as a "rigid" frame. It can be shown that as a consequence the acceleration  $\mathbf{a}$  is not the same for the different points of  $S$ , but we are only interested in points inside a small neighborhood of the accelerated object [Rindler (1991)]. Specifically we are interested in a neighborhood of the object's central point that contains the object and within which the acceleration is everywhere essentially the same.

Because of the hyperbolic motion, the velocity  $u_x(\tau) = c\beta_\tau$  in  $S$  with respect to  $I_*$ , is

$$\beta_\tau = \frac{u_x(\tau)}{c} = \tanh\left(\frac{a\tau}{c}\right) \quad (1)$$

and then

$$\gamma_\tau = (1 - \beta_\tau^2)^{-1/2} = \cosh\left(\frac{a\tau}{c}\right) \quad (2)$$

The ZPF in the laboratory system  $I_*$  is given by

$$E^{zp}(\bar{\mathbf{R}}_*, t_*) = \sum_{\lambda=1}^2 \int d^3k \hat{\mathbf{e}}(k, \lambda) H_{zp}(\omega) \cos[\bar{\mathbf{k}} \cdot \bar{\mathbf{R}}_* - \omega t_* - \theta(\bar{\mathbf{k}}, \lambda)], \quad (3a)$$

$$B^{zp}(\bar{\mathbf{R}}_*, t_*) = \sum_{\lambda=1}^2 \int d^3k (\hat{\mathbf{k}} \times \hat{\mathbf{e}}) H_{zp}(\omega) \cos[\bar{\mathbf{k}} \cdot \bar{\mathbf{R}}_* - \omega t_* - \theta(\bar{\mathbf{k}}, \lambda)]. \quad (3b)$$

$\bar{\mathbf{R}}_*$  and  $t_*$  refer respectively to the space and time coordinates of the point of observation of the field in  $I_*$ . At  $t_* = 0$ , the point  $\bar{\mathbf{R}}_* = (c^2/a)\hat{x}$  of  $I_*$  and the particle in  $S$  coincide. The phase term  $\{\theta(\mathbf{k}, \lambda)\}$  is a family of random variables, uniformly distributed between 0 and  $2\pi$ , whose mutually independent elements are indexed by the wavevector  $\mathbf{k}$  and the polarization index  $\lambda$ . Furthermore one defines,

$$H_{zp}^2(\omega) = \frac{\hbar\omega}{2\pi^2}. \quad (4)$$

The coordinates  $\bar{\mathbf{R}}_*$  and time  $t_*$  refer to the particle point of the accelerated frame  $S$  as viewed from  $I_*$ . We, for convenience, Lorentz-transform the fields from  $I_*$  to the corresponding  $I_\tau$  frame tangential to  $S$  and then, omitting for simplicity to display explicitly the  $\lambda$  and  $\mathbf{k}$  dependence in the polarization vector

$$\hat{\varepsilon} = \hat{\varepsilon}(\vec{k}, \lambda),$$

we obtain

$$\bar{E}^{\mathcal{P}}(0, \tau) = \sum_{\lambda=1}^2 \int d^3k \left\{ \hat{x} \hat{\varepsilon}_x + \hat{y} \gamma_{\tau} \left[ \hat{\varepsilon}_y - \beta_{\tau} (\hat{k} \times \hat{\varepsilon})_z \right] + \hat{z} \gamma_{\tau} \left[ \hat{\varepsilon}_z + \beta_{\tau} (\hat{k} \times \hat{\varepsilon})_y \right] \right\} \times H_{\mathcal{P}}(\omega) \cos[\vec{k} \cdot \bar{R}_* - \omega t_* - \theta(\vec{k}, \lambda)] \quad (5a)$$

$$\bar{B}^{\mathcal{P}}(0, \tau) = \sum_{\lambda=1}^2 \int d^3k \left\{ \hat{x} (\hat{k} \times \hat{\varepsilon})_x + \hat{y} \gamma_{\tau} \left[ (\hat{k} \times \hat{\varepsilon})_y + \beta_{\tau} \hat{\varepsilon}_z \right] + \hat{z} \gamma_{\tau} \left[ (\hat{k} \times \hat{\varepsilon})_z - \beta_{\tau} \hat{\varepsilon}_y \right] \right\} \times H_{\mathcal{P}}(\omega) \cos[\vec{k} \cdot \bar{R}_* - \omega t_* - \theta(\vec{k}, \lambda)], \quad (5b)$$

where the zero in the argument of the  $I_{\tau}$  fields,  $\mathbf{E}^{\mathcal{P}}$  and  $\mathbf{B}^{\mathcal{P}}$  actually means the  $I_{\tau}$  spatial point  $(c^2/a, 0, 0)$ . Here we observe that we take the fields that correspond to the ZPF as viewed from every inertial frame  $I_{\tau}$  (whose  $(c^2/a, 0, 0)$  point coincides with the particle point  $(c^2/a, 0, 0)$  of  $S$  and instantaneously comoves with the particle at the corresponding instant of proper time  $\tau$ ), to also represent the ZPF viewed instantaneously and from the single point,  $(c^2/a, 0, 0)$  in  $S$ .

We can select space and time coordinates and orientation in  $I_*$  such that

$$\bar{R}_*(\tau) \cdot \hat{x} = \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) \quad (6)$$

$$t_* = \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right) \quad (7)$$

From the equations above one obtains [1, 4]

$$\begin{aligned} \bar{E}^{\mathcal{P}}(0, \tau) = & \sum_{\lambda=1}^2 \int d^3k \\ & \times \left\{ \hat{x} \hat{\varepsilon}_x + \hat{y} \cosh\left(\frac{a\tau}{c}\right) \left[ \hat{\varepsilon}_y - \tanh\left(\frac{a\tau}{c}\right) (\hat{k} \times \hat{\varepsilon})_z \right] + \hat{z} \cosh\left(\frac{a\tau}{c}\right) \left[ \hat{\varepsilon}_z + \tanh\left(\frac{a\tau}{c}\right) (\hat{k} \times \hat{\varepsilon})_y \right] \right\} \\ & \times H_{\mathcal{P}}(\omega) \cos \left[ k_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \frac{\omega c}{a} \sinh\left(\frac{a\tau}{c}\right) - \theta(\vec{k}, \lambda) \right] \end{aligned} \quad (8a)$$

$$\begin{aligned}
\bar{B}_\tau^{zp}(0, \tau) = & \sum_{\lambda=1}^2 \int d^3k \\
& \times \left\{ \hat{x}(\hat{k} \times \hat{e})_x + \hat{y} \cosh\left(\frac{a\tau}{c}\right) \left[ (\hat{k} \times \hat{e})_y + \tanh\left(\frac{a\tau}{c}\right) \hat{e}_z \right] + \hat{z} \cosh\left(\frac{a\tau}{c}\right) \left[ (\hat{k} \times \hat{e})_z - \tanh\left(\frac{a\tau}{c}\right) \hat{e}_y \right] \right\} \\
& \times H_{zp}(\omega) \cos \left[ k_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \left(\frac{\omega c}{a}\right) \sinh\left(\frac{a\tau}{c}\right) - \theta(\bar{k}, \lambda) \right]
\end{aligned} \tag{8b}$$

This is the ZPF as instantaneously viewed from the particle fixed to the point  $(c^2/a, 0, 0)$  of S that is performing the hyperbolic motion.

### INERTIA REACTION FORCE AND THE ZPF MOMENTUM DENSITY

First we consider the following simple fluid analogy involving as a heuristic device a constant velocity and a spatially varying density in place of the usual hyperbolic motion through a uniform vacuum medium. Let a small *geometric figure* of a fixed proper volume  $V_0$ , move uniformly with *subrelativistic* velocity  $v$  along the  $x$ -direction. The volume  $V_0$  we imagine as always immersed in a fluid that is isotropic, homogeneous and at rest, except such that its density  $\rho(x)$  increases in the  $x$ -direction but is uniform in the  $y$ - and  $z$ -directions. Hence, as this small fixed volume  $V_0$  moves in the  $x$ -direction, the mass enclosed in its volume,  $V_0\rho(x)$ , increases. In an inertial frame at rest with respect to the geometric figure the mass inside the volume  $V_0\rho(x)$ , is seen to grow. Concomitantly it is realized that the volume  $V_0$  is sweeping through the fluid and that this  $V_0\rho(x)$  mass grows because there is a *net influx* of mass coming into  $V_0$  in a direction opposite to the direction of the velocity  $v$ . In an analogous fashion, for the more complex situation envisaged in this paper, simultaneously with the steady growth of the ZPF momentum contained within the volume of the object discussed above, the object is sweeping through the ZPF of the  $I_+$  inertial observer and for him there is a net influx of momentum density coming from the background into the object and in a direction opposite to that of the velocity of the object.

As it is the ZPF radiation background of  $I_+$  in the act of being swept through by the particle which we are calculating now, we fix our attention on a fixed point of  $I_+$ , say the point of the observer at  $(c^2/a, 0, 0)$  of  $I_+$ , that momentarily coincides with the object at the object proper time  $\tau = 0$ , and consider that point as referred to the inertial frame  $I_\tau$  that instantaneously will coincide with the object at a future generalized object proper time  $\tau > 0$ . Hence we compute the  $I_\tau$ -Poynting vector, but evaluated at the  $(c^2/a, 0, 0)$  space point of the  $I_+$  inertial frame, namely in  $I_\tau$  at the  $I_\tau$  space-time point:

$$ct_\tau = -\frac{c^2}{a} \sinh\left(\frac{a\tau}{c}\right), \tag{9}$$

$$x_\tau = \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right), y_\tau = 0, z_\tau = 0, \tag{10}$$

where  $t_\tau$ , the time of  $I_\tau$  is selected such that  $t_\tau = 0$  at proper time  $\tau$  when the particle comoves and coincides with the  $(c^2/a, 0, 0)$  point of  $I_\tau$ . This Poynting vector we shall denote by  $N^{zp}$ . Everything however is ultimately referred to the  $I_+$  inertial frame as that is the frame of the observer that looks at the object and whose ZPF background the moving object is sweeping through. In order to accomplish this we first compute

$$\begin{aligned}
\langle \bar{E}_\tau^{zp}(0, \tau) \times \bar{B}_\tau^{zp}(0, \tau) \rangle_x &= \langle E_{y\tau} B_{z\tau} - E_{z\tau} B_{y\tau} \rangle \\
&= \gamma_\tau^2 \langle (E_{y*} - \beta_\tau B_{z*})(B_{z*} - \beta_\tau E_{y*}) - (E_{z*} + \beta_\tau B_{y*})(B_{y*} + \beta_\tau E_{z*}) \rangle \\
&= -\gamma_\tau^2 \beta_\tau \langle E_{y*}^2 + B_{z*}^2 + E_{z*}^2 + B_{y*}^2 \rangle + \gamma_\tau^2 (1 + \beta_\tau^2) \langle E_{y*} B_{z*} - E_{z*} B_{y*} \rangle \\
&= -\gamma_\tau^2 \beta_\tau \langle E_{y*}^2 + B_{z*}^2 + E_{z*}^2 + B_{y*}^2 \rangle
\end{aligned} \tag{11}$$

that we use in the evaluation of the Poynting vector

$$\bar{N}_*^{zp} = \frac{c}{4\pi} \langle \bar{E}_\tau^{zp} \times \bar{B}_\tau^{zp} \rangle_* = \hat{x} \frac{c}{4\pi} \langle E_\tau^{zp}(0, \tau) \times B_\tau^{zp}(0, \tau) \rangle_x \tag{12}$$

The integrals are now taken with respect to the I- ZPF background as that is the background that the I-observer considers the object to be sweeping through. This is why we will denote this Poynting vector as  $N_*^{zp}$ , with an asterisk subindex instead of a  $\tau$  subindex, to indicate that it refers to the ZPF of I-. Observe that in eq.(11) the term proportional to the ordinary ZPF Poynting vector of I-, vanishes. The net amount of momentum of the background the particle has swept through after a time  $t_*$ , as judged again from the I\* frame viewpoint, is

$$\bar{p}_*^{zp} = \bar{g}_*^{zp} V_* = \frac{\bar{N}_*^{zp}}{c^2} V_* = -\hat{x} \frac{1}{c^2} \frac{c}{4\pi} \gamma_\tau^2 \beta_\tau \frac{2}{3} \langle \bar{E}_*^2 + \bar{B}_*^2 \rangle V_* \tag{13}$$

We can compute Eqs. (12) and (13) in more detail. This as well as many other details on the analysis will appear elsewhere (Rueda and Haisch, 1997). The Poynting vector that the radiation should have at the  $(c^2/a, 0, 0)$  point of I- but referred to I- with the coordinates of eq.(11), can be shown to be

$$\bar{N}_*^{zp}(\tau) = \frac{c}{4\pi} \langle \bar{E}^{zp} \times \bar{B}^{zp} \rangle = \frac{c}{4\pi} \hat{x} \langle E_y B_z - E_z B_y \rangle = -\frac{c}{4\pi} \frac{8\pi}{3} \left[ \int \frac{\hbar \omega^3 d\omega}{2\pi^2 c^3} \right] \sinh\left(\frac{2a\tau}{c}\right) \hat{x} \tag{14}$$

where  $\mathbf{E}$  and  $\mathbf{B}$  stand for  $\mathbf{E}_\tau(0, \tau)$  and  $\mathbf{B}_\tau(0, \tau)$  respectively as in the case of eq.(12) and where as in eqs.(11), (12) and (13) the integration is understood to proceed over the  $k$ -sphere of I-. The particle now is not in uniform but instead in accelerated motion. If suddenly, at proper time  $\tau$ , the motion were to switch from hyperbolic back to uniform because the accelerating action disappeared, we would just need to replace in eq.(14) the constant rapidity  $\rho$  at that instant for  $a\tau$ , and  $\beta_\tau$  in eq. (1) would then become  $\tanh(\rho/c)$ . (But then  $N_*^{zp}$  would cease to be, for all times onward, a function of  $\tau$  and force expressions as eq.(17) below would vanish). Observe that we make explicit the  $\tau$  dependence of this as well as of the subsequent quantities below.  $N_*^{zp}(\tau)$  represents energy flux, i.e., energy per unit area and per unit time in the  $x$ -direction. It also implies a parallel,  $x$ -directed momentum density, i.e., field momentum per unit volume incoming towards the particle position,  $(c^2/a, 0, 0)$  of S, at particle proper time  $\tau$  and as estimated from the viewpoint of I-. Explicitly such momentum density is

$$\bar{g}_*^{zp}(\tau) = \frac{\bar{N}_*^{zp}(\tau)}{c^2} = -\hat{x} \frac{8\pi}{3} \frac{1}{4\pi c} \sinh\left(\frac{2a\tau}{c}\right) \int \eta(\omega) \frac{\hbar \omega^3}{2\pi^2 c^3} d\omega, \tag{15}$$

where we now introduce the frequency-dependent coupling coefficient,  $0 \leq \eta(\omega) \leq 1$ , that quantifies the fraction of absorption or scattering at each frequency. Let  $V_0$  be the proper volume of the particle, namely the volume that the particle has in the reference frame  $I_r$  where it is instantaneously at rest at proper time  $\tau$ . From the viewpoint of  $I_*$ , however, such volume is then  $V_* = (1/\gamma_\tau)V_0$  because of Lorentz contraction. The amount of momentum due to the radiation inside the volume of the particle according to  $I_*$ , i.e., the radiation momentum in the volume of the particle viewed at the laboratory is

$$\bar{p}_*{}^{ip} = V_* \bar{g}_*{}^{ip}(\tau) = \frac{V_0}{\gamma_\tau} \bar{g}_*{}^{ip}(\tau) = -\hat{x} \frac{4V_0}{3} (c\beta_\tau) \gamma_\tau \left( \frac{1}{c^2} \int \eta(\omega) \frac{\hbar \omega^3 d\omega}{2\pi^2 c^3} \right) \quad (16)$$

which is again eq. (13).

At proper time  $\tau = 0$ , the  $(c^2/a, 0, 0)$  point of the laboratory inertial system  $I_*$  instantaneously coincides and comoves with the particle point of the Rindler frame  $S$  in which the particle is fixed. The observer located at  $x_* = c^2/a$ ,  $y_* = 0$ ,  $z_* = 0$  instantaneously, at  $t_* = 0$ , coincides and comoves with the particle but because the latter is accelerated with constant acceleration  $\mathbf{a}$ , the particle according to  $I_*$  should receive a time rate of change of incoming ZPF momentum of the form:

$$\frac{d\bar{p}_*{}^{ip}}{dt_*} = \frac{1}{\gamma_\tau} \frac{d\bar{p}_*{}^{ip}}{d\tau} \Big|_{\tau=0} \quad (17)$$

We *postulate* that such rate of change may be identified with a force from the ZPF on the particle. Such interpretation, intuitively at least, looks natural. If the particle has a proper volume  $V_0$ , the force exerted on the particle by the radiation from the ZPF as seen in  $I_*$  at  $t_* = 0$  is then

$$\frac{d\bar{p}_*{}^{ip}}{dt_*} = \bar{f}_*{}^{ip} = - \left( \frac{4}{3} \frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar \omega^3 d\omega}{2\pi^2 c^3} \right) \bar{\mathbf{a}}_* = -m_i \bar{\mathbf{a}}_* \quad (18)$$

Furthermore

$$m_i \equiv \left( \frac{V_0}{c^2} \int \frac{\eta(\omega) \hbar \omega^3 d\omega}{2\pi^2 c^3} \right) \quad (19)$$

is an invariant scalar with the dimension of mass. Observe that in eq.(19) we have neglected a factor of 4/3 that should appear multiplying in front. Such factor must be neglected because a fully covariant analysis (Rueda and Haisch, 1997) shows that it disappears. The corresponding form of  $m_i$  as written (and without the 4/3 factor) is susceptible of a natural interpretation: Inertial mass is the mass of the energy of the ZPF radiation enclosed within the particle and that does actually interact with it ( $\eta(\omega)$  factor in the integrand).

## THE ZERO-POINT FIELD MOMENTUM CONTENT

Limitations in the space prevents us from discussing an important complementary approach to the previous one. The corresponding analysis is however similar to that above and will be displayed in Rueda and Haisch (1997). It produces instead of the time rate of  $\bar{p}^{ip}$ , the time rate of  $\bar{p}_*$ , the momentum content of the particle. The analysis

yields a natural interpretation. The following feature deserves special attention. After the acceleration process is completed, from the point of view of an inertial observer attached to the stationary laboratory frame there appears associated with the body in motion a net flux of momentum density in the surrounding ZPF. In other words, on calculating the ZPF momentum contained in the object as referenced to the observer's own inertial frame, the observer would conclude that a certain amount of momentum is instantaneously contained within the proper volume  $V_0$  of the moving object. This momentum is directly related to what would normally be called the physical momentum of the object. Calculated with respect to its own frame the object itself would find not net ZPF momentum contained within itself, consistent with the view that one's own momentum is necessarily always zero.

## RELATIVISTIC FORCE EXPRESSION

From the definition of the momentum  $\vec{p}$  in eq.(16), from eqs.(17), (18), and Newton's third law it immediately follows that the momentum of the particle is

$$\vec{p} = m \gamma_r c \vec{\beta}_r \quad (20)$$

in exact agreement with the momentum expression for a moving particle in special relativity. The expression for the space vector component of the four-force is then

$$\vec{F}_r = \gamma_r \frac{d\vec{p}_r}{d\tau} = \frac{d\vec{p}_r}{d\tau} \quad (21)$$

and as the force is *pure* in the sense of Rindler (1991), after dropping the  $r$  subindex the correct form for the four-force immediately follows

$$\mathcal{F} = \frac{d\mathcal{P}}{d\tau} = \frac{d}{d\tau}(\gamma_r m_r c, \vec{p}_r) = \gamma_r \left( \frac{1}{c} \frac{dE}{dt}, \vec{f}_r \right) = \gamma_r (\vec{f}_r \cdot \vec{\beta}_r, \vec{f}_r) = (\vec{F}_r \cdot \vec{\beta}_r, \vec{F}_r), \quad (22)$$

in the ordinary way anticipated above.

## CONCLUSION

As the expression for the ZPF reaction force of eq (18) depends only on the instantaneous value of the acceleration imposed on the accelerated object by the accelerating agent, it arguably follows that this absence of any memory effects, i.e., of any expression in the force that reveals its underlying unidirectional hyperbolic motion origin, permits to readily generalize the argument to much more general type of motions. Further relevant features of the general argument (Rueda and Haisch, 1997) not mentioned here are a fully covariant calculation, a discussion and analysis of the character of the k-space integrations and a more detailed evaluation of the Poynting cross products. These will also be presented in Rueda and Haisch (1997).

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